Shocks on Public Spending and Economic Growth in Fragile States

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Abstract: This paper focuses on fragile States, and look deeply how shocks on public spending affect private production, economic growth and households' welfare. The paper provides an explanation of a source of growth and technological progress in unstable countries. The increasing of public expenditures enhances the private production and households' consumption. One innovation of this paper is in the way to introduce shocks into economy. In fragile States, shocks are random variables that follow a Bernoulli process, which appear on public spending and affect the rest of economy.

Keywords: Fragile States, public spending, economic growth, shocks, Bernoulli process,

Introduction

DFID defines fragile States as: "those where the government cannot or will not deliver core functions to the majority of its people, including the poor" (DFID[1], 2005). Commonly, fragile States are described as incapable to assume basic security, maintain rule of law and justice, or provide basic services and economic opportunities for their citizens. Fragile State are mostly developing countries, facing violence and conflict, political instability, severe poverty, and other threats to security and development (Nay O., 2013). In fragile States, government survival and public spending are highly uncertain. However, well-directed public spending can enhance the welfare of populations.

Indeed, the first decade of the 21st century ended with financial crises similar to that of 1929 in US, with economic consequences that required systematic and massive State intervention. Thus, during 2009, the American federal government injected great sums of money to save major sectors of the economy such as the automobile, energy, research and development, etc.

Stiglitz (2003) goes further to show that beyond received ideas, the federal state has often been directly or indirectly interventionist in vital sectors of the American economy. During 2010, the countries of the European Union practiced expansionist policies, and carried out partial nationalizations of banks and industrial companies. Thus, public spending has explicitly played a counter-cyclical role in most OECD countries. Optimal public expenditure is the one that maximizes efficiency and allows the Government to fulfill its allocation and stabilization functions. However, the roles of automatic stabilizers and budget recovery are often questioned by the new classical school and the economic school of the offer due to the risks of inflation.

Beyond traditional theoretical controversies between new classic and Keynesian schools, different empirical approaches are made. Relations between public spending and growth are discussed in terms of quality of public spending by Afonso and al. (2005), State size and public spending by Barro (1990), and Afonso and Furceri (2010), the nature of spending by Varoudakis (1996) and N'Gouan (2011), business cycle by Backus and al. (1995), a mode of financing public spending and deficits by the monetarist school, ect. All these studies globally prove the positive impact of public spending on growth.

How to capture the impact of shocks on public spending on the whole economy? Some studies attempt to answer the question using calibration methods (Baxter and King, 1993), vector autoregressive (VAR) models (Backus and al. 1995).

Cimadomo and al. (2011) identify the effects of government spending shocks with and without expected reversal using an approach based on U.S. real-time data. Based on a structural VAR analysis, their results suggest that shocks associated with an expected spending reversal exert expansionary effects on the economy. Shocks associated with expected spending growth above trend, instead, are characterized by a contraction in aggregate demand.

Ramey and Shapiro (1998) analyze the effects of sector-specific changes in government spending in a two-sector dynamic general equilibrium. The empirical part of their paper estimates the effects of military buildups on a variety of macroeconomic variables using a new measure of military shocks. They found that

behavior of macroeconomic aggregates is consistent with the predictions of a multi-sector neoclassical model. The authors show that, the effects on output and hours may be magnified, interest rates may fall, and consumption and real wage may fall.

In standard VAR identification approaches, shocks on government spending raise consumption and real wage, whereas the Ramey and Shapiro (1998) narrative identification approach shocks on government spending lower consumption and real wages. Ramey (2011) show that a key difference in the approaches is the timing. Simulations from a standard neoclassical model in which government spending is anticipated by several quarters demonstrate that, VARs estimated with faulty timing can produce a rise in consumption even when it decreases in the model. Ramey (2011) also shows that most components of consumption fall after a positive shock to government spending and that the implied government spending multipliers are ranged from 0.6 to 1.1.

Olumide O. and al. (2020) examine the asymmetric effect of government spending on economic growth in Nigeria over the period 1980-2017. The authors find that the response of economic growth to government spending shocks differs according to the nature of shocks on them. More specifically, their study established that the stabilizing effects of fiscal policies are dependent on the state of the business cycle.

Jorgensen and Ravn (2022) study the inflation response to government spending shocks. They present empirical evidence, that prices do not increase in response to a positive government spending shock. Their study contrast with standard new Keynesian models predict that expansionary fiscal policy is inflationary.

The aim of this study is to examine how shocks to public expenditure affect private production, economic growth and household welfare. We do not use VARs models. We are building a dynamic general equilibrium model, specific to fragile states. This study is more suited to fragile countries. Those countries are prone to conflict, insecurity, poverty, etc. In fragile states, the future is sometimes uncertain and economic forecasts often non-existent. No one knows in advance if the government will make productive public spending, if the government will create new roads, or open public hospitals, etc. Public spending is therefore uncertain. In this paper, we reformulate uncertain public spending as shocks, which can affect economic growth and the welfare of populations.

In this study, the shocks are discrete random variables, which follow a Bernoulli process, which appear on public spending and affect the rest of the economy. Through

shocks on public spending, we explain a potential source of economic growth. We shed new light on what economists often call technological progress[2]. We describe a small closed economy, in which the long-run growth rate is dynamic. This growth rate can decrease or increase according to shocks on public spending.

Such economy is typical of fragile States, where the level of public spending is uncertain. In fragile states, the level of public expenditure depends on uncertain resources, such as the volume of donations and external aid received, climatic changes, political instability, debt service, etc. This document is divided into two parts. The first part shows how an increase in government spending leads to an increase in output. The second part deals with household behavior. We show that consumption of households and their welfare depend entirely on the history of shocks on publics spending.

1.   The production sector and role of public spending

Let's consider an economy with a private production sector, which uses private inputs: capital K and labor L, to produce single final consumption good Y. Public expenditures[3] are not known in advance, they are uncertain until their realization. Public spending appears as externalities for the private production sector and enter directly as an input into the production of the final consumption good Y. We assume that private sector compagnies are all identical, they use the same technology to produce the same good. The number of firms is normalized to 1, That's to assume a single representative company that produces for the whole economy. We adopt for a classical approach to public goods: public goods are non-rival and non-exclusive. Public spending, when it is carried out, is beneficial to all economic agents. Each private company benefits from all public expenditure; the use of public expenditure by one firm does not decrease the amount available to other firms. Following Barro (1990), we assume that the production function[4] of the private sector is of Cobb-Douglas[5].

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} G_t^{1-\alpha} \tag{1}$$

Where α, $0 < \alpha < 1$ is the capital intensity; Y_t denotes the quantity of consumption good produced by the private sector at date t; G_t denotes the level of public expenditures. The production function of the private sector has a constant return to scale with respect to the private factors of production: labor L and capital K.

We assume the choice of the level of public expenditure is entirely determined by the hazardous and discretionary decisions of the government. The government

levies taxes on households income, and buys part of the private production. The government then uses these purchases to provide free services to private producers. Government spending directly influences the level of production in the private sector. Hazardous public spending choices are made at the beginning of each year by the government. Nevertheless, the government has the power to increase or reduce public expenditure. Public expenditures, unknown in advance to the private sector, behave like shocks. Public spending G boosts or weakens private sector production. The index t denotes the time, assumed to be discrete. In formulation (1), an increase in public spending boosts the marginal productivity of labor *L* and capital *K*.

Labor supply is provided by households. The population size N_t grows at a constant rate *n*, there is no unemployment. At date *t* = 0, the size of the population is fixed at N_0 = 1. Households do not value leisure. Since unemployment is assumed to be non-existent, the labor supply in equilibrium grows at the same rate as the population:

$$L_t = N_t = (1+n)^t \qquad (2)$$

Unlike Barro's model[7], the share of public expenditure in production is not constant. The amount of public expenditure G_t depends on the whole history of discretionary and hazardous decisions of the government at the beginning of each period *t*.

$$G_t = \begin{cases} uG_{t-1}\,; & with\ the\ probability P \\ dG_{t-1}\,; & with\ the\ probability\ 1-P \end{cases} \qquad (3.a)$$

Whith[8], $0 < d < 1 < u$, thus, depending on the hazardous decision of the government - unknown in advance, the size of public expenditure increases with a probability *P*, or decreases with a probability $1 - P, 0 \leq P \leq 1$.

However, the government balances its budget by levying a variable rate tax[9] τ_t on gross households' income: $G_t = \tau_t \cdot Y_t$. The amount of government expenditure in expression (3.a) can be rewritten as follows:

$$G_t = u^{X_t} \cdot d^{1-X_t} \cdot G_{t-1} \qquad (3.b)$$

Where, X_t is a random variable that follows a Bernoulli law. X_t Takes the value 1 with the probability *P*, and the value 0 with the probability 1-*P*. The variables indexed by *t*, are all measurable[10]; they can be written as a function of shocks history. By developing expression (3.b), we obtain:

$$G_t = u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0 \qquad (3.c)$$

Expression (3.c) makes clear that, the amount of government spending depends on the whole history of government hazardous spending decisions. Substituting (3.c) in expression (1) and remember that in equilibrium the supply of employment is equal to the demand for employment, we find that the production function is written by:

$$Y_t = A_t^{1-\alpha} \cdot K_t^{\alpha} \cdot N_t^{1-\alpha}$$

Where[11], $A_t = u^{\Sigma_{j=1}^{t} X_j} \cdot d^{t-\Sigma_{j=1}^{t} X_j} \cdot G_0$, reflects technological progress (or fall), induced by public spending; G_0 is given. A_t sheds new light on what economists have called technological progress. Indeed, the production factors (labor and capital) are not enough to explain economic growth. A(t) describes the impact of public expenditures[12] on output. The source of technological progress or backwardness is the public spending decisions.

In fact, if the government chooses to reduce public expenditures from year to year ($X_t = 0$, $t = 1,2,\ldots$) then, since $0 < d < 1$, the production of the private sector will decrease over time, $Y_t = (d^t \cdot G_0)^{1-\alpha} \cdot K_t^{\alpha} \cdot N_t^{1-\alpha}$. Furthermore, for a fixed level of capital, if population growth does not compensate for the reduction in public spending ($1 + n \geq d$), private sector output will become lower and lower, and in the long run will be close to 0. The populations will find themselves in poverty and generalized misery. If, on the other hand, the government chooses to increase public spending from year to year, then private sector output will be strongly stimulated by public spending. In this context: $Y_t = (u^t \cdot G_0)^{1-\alpha} \cdot K_t^{\alpha} \cdot N_t^{1-\alpha}$, with $u > 1$. Thus, the budgetary decisions of the government directly affect the production of the private sector.

The graph below is the result of a Principal Component Analysis[13] carried out on 106 countries, with data for the year 2019, provided by World Development Indicators[14]. The retained variables for each country are: GDP per capita, government expenditure per capita, final consumption goods per capita, government income per capita and total government expenditure per capita. Expenditure per capita, income per capita are variables that strongly contributed to the formation of the first factor. The second factor is GDP per capita.

In this exploratory analysis, we observe that the so-called developed countries of OECD generally coincide with the countries where per capita government expenditure is high. In OECD countries, GDP per capita is higher than the average GDP of the countries studied. On the other hand, the so-called developing countries

of Africa and Asia are generally those where public expenditure and per capita GDP are low. This principal component analysis supports the idea that the increasing in public spending boosts the level of GDP per capita.
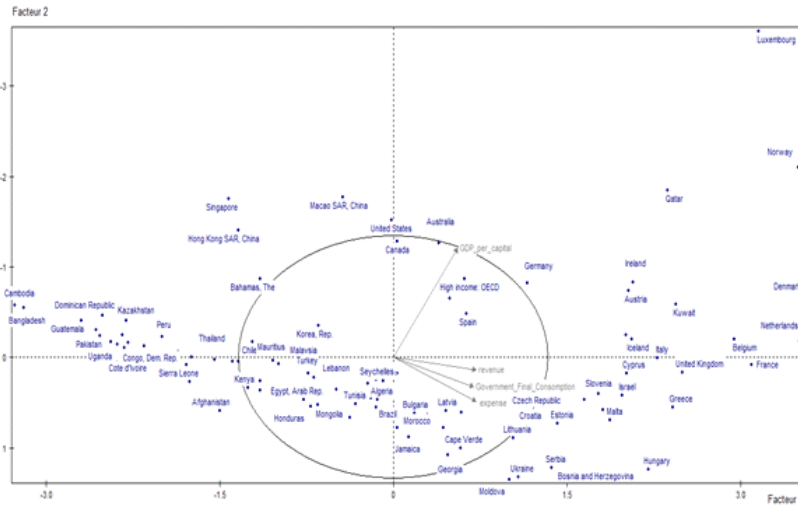

Chart: Descriptive analysis of countries

Data source: World Bank, World development Indicators, 2019. Website: Banque Mondiale, World Development Indicators, 2019. Website: http://data.worldbank.org/data-catalog/world-development-indicators.

At each instant *t*, the problem of the firms which produce the consumption good is to choose the quantity of labor L_t and of capital K_t which maximizes their profit. The profit of the firm is written by:

$$\pi_t = A_t^{1-\alpha} \cdot K_t^{\alpha} \cdot L_t^{1-\alpha} - w_t \cdot L_t - r_t \cdot K_t$$

Where w_t is the real wage rate, i.e. the quantity of unit of final good that the firm gives in exchange for a unit of work. r_t is the real rental price of capital, it is the number of units of final good that should be given in exchange for one unit of capital. The consumption good Y_t is therefore chosen as numerary. The first-order conditions for the firm problem imply:

$$r_t = \alpha \cdot \frac{Y_t}{K_t} = \alpha \cdot A_t^{1-\alpha} \cdot K_t^{\alpha-1} \cdot L_t^{1-\alpha}$$

$$w_t = (1-\alpha) \cdot \frac{Y_t}{L_t} = (1-\alpha) \cdot A_t^{1-\alpha} \cdot K_t^{\alpha-1} \cdot L_t^{1-\alpha}$$

Thus, at equilibrium, the firm's profit is zero: $\pi_t = Y_t - w_t \cdot L_t - r_t \cdot K_t = 0$.

2. Behavior of households and their welfare and steady state

Households receive a wage w_t as counterpart for their work N_t. The Capital belongs to households. Households rent capital to films. The capital owned by households is remunerated at the rate r_t. All markets are open on each date t. Households use their income (net of tax) from work and interest to buy consumption good Y. After-tax household income is written as:

$$(1 - \tau_t) \cdot (w_t \cdot L_t + r_t \cdot K_t) = (1 - \tau_t) \cdot Y_t$$

Where, $\tau_t = \frac{G_t}{Y_t} = \frac{T_t}{Y_t}$ is the tax levy rate. τ_t is not a choice variable for companies, nor for households. τ_t is a discretionary government choice variable. τ_t is the tax rate that balances the government budget. By using relation (3.b), we obtain[15] :

$$\tau_t = \frac{u^{X_t} \cdot d^{1-X_t}}{1 + \gamma_t} \tau_{t-1}$$

Where $\gamma_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$, is the growth rate of the output. Thus, the tax rate increases if the government chooses to increase public spending and vice versa.

We assume that all households are identical and each household has a unit of Labor. Each household includes one adult who participate in the production of the final good. Although each individual's life is finite, we suppose an immortal extended family. The household problem at each instant t ? 0, is to maximize the intertemporal utility function, defined by:

$$E_t \left[\sum_{s=0}^{+\infty} \beta^{t+s} \cdot U(C_{t+s}) \right] \tag{4}$$

Where: β, $0 < \beta < 1$ is the discount factor, β is the depreciation rate of the future, C_t is the level of consumption goods at time t. Instantaneous utility:

$$U(C) = \frac{C^{1-\sigma}}{1 - \sigma}$$

$U(C)$ is a Borel function, differentiable, strictly increasing, concave. σ, $0 < \sigma \leq 1$, is the intertemporal elasticity of substitution of consumption. When[16] $\sigma = 1$, the instantaneous utility has a logarithmic form. Expression (4) shows that the global utility function, (the goal) of (infinite-lived) households, is the weighted sum of all

present and future flows of elementary utilities. E_t denotes the conditional expectation on the information available at date t. The households' constraints are written as:

$$(1 - \tau_t) \cdot Y_t = C_t + I_t \tag{5.a}$$

$$Y_t = w_t \cdot L_t + r_t \cdot K_t = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} \tag{5.b}$$

$$K_{t+1} = I_t + (1 - \delta)K_t \tag{5.c}$$

Where, I_t is the level of investment; δ, $0 < \delta < 1$ is the depreciation rate of capital; K_t is the stock of capital available at the date t. Constraint (5.a) reflects the fact that aggregate household income (net of tax) is divided between consumption and investment. Constraint (5.c) describes capital accumulation. (5.b) is an identity, which describes the equality between aggregate supply and aggregate demand. By combining the constraints (5.a), (5.b) and (5.c), we obtain at each date *t*, households resource constraint:

$$(1 - \tau_t) \cdot Y_t = C_t + K_{t+1} - (1 - \delta)K_t \tag{5.d}$$

Households' problem at each date *t*, is therefore to choose the level of consumption C_t and future capital K_{t+1}, which maximize expression (4), under the resource constraint (5.d). To solve this problem, we redefine the variables per efficient unit. Let's say:

$$\widehat{N}_t = A_t . N_t$$

$\widehat{N}_t$ is interpreted as efficient labor. $\widehat{N}_t$ introduces into labor the effects of public spending. We also define production, consumption and capital per efficient unit by:

$$\hat{y}_t = \frac{Y_t}{\widehat{N}_t} = \hat{k}_t^{\alpha} \quad ; \quad \hat{c}_t = \frac{C_t}{\widehat{N}_t} \quad ; \quad \hat{k}_t = \frac{K_t}{\widehat{N}_t}$$

The resource constraint can now be rewritten as follows:

$$(1 - \tau_t) \cdot \hat{k}_t^{\alpha} = \hat{c}_t + (1 + n) \cdot u^{X_{t+1}} \cdot d^{1-X_{t+1}} \cdot \hat{k}_{t+1} - (1 - \delta)\hat{k}_t \tag{6.a}$$

Under the constraints (6.a), households' problem can be formulated as:

$$\max_{\hat{c}_{t+s}, \ \hat{k}_{t+s}} E_t \left[\sum_{s=0}^{+\infty} \tilde{\beta}^s \frac{\hat{c}_{t+s}^{1-\sigma}}{1 - \sigma} \right] \tag{6.b}$$

The advantage of carrying out a transformation of the variables is twofold. On the one hand, the transformation makes it possible to take an interest in the problem of each household. The variables per efficient unit are per capita variables, where the impact of public expenditure is introduced. Thus, the change of variables

transforms the global household's problem to the problem of each individual. On the other hand, the transformation of the variables makes it possible to work with a variable discount factor, which takes into account the size of the population and the impact of public expenditure: $\tilde{\beta}^t = \beta^t \cdot (1+n)^{(1-\sigma)t} \cdot A_t.$.

The Lagrangian for the households' problem at each date t is written as:

$$L_t = E_t \sum_{s=0}^{+\infty} \left\{ \tilde{\beta}^s \frac{\hat{c}_{t+s}^{1-\sigma}}{1-\sigma} + \right.$$

$$\mu_{t+s} \left[(1-\tau_{t+s}) \cdot \hat{y}_{t+s} - \hat{c}_{t+s} - (1+n) \cdot u^{X_{t+1+s}} \cdot d^{1-X_{t+1+s}} \cdot \hat{k}_{t+1+s} + (1-\delta)\hat{k}_{t+s} \right] \right\}$$

Where μ_{t+s} is the Lagrange multiplier. The first-order conditions are obtained by differentiating the Lagrangian with respect to $\hat{c}_{t+s}$, $\hat{k}_{t+s}$ and μ_{t+s}.

$$E_t \left[\tilde{\beta}^s \hat{c}_{t+s}^{-\sigma} - \mu_{t+s} \right] = 0$$

$$E_t \left[-\mu_{t+s}(1+n) \cdot u^{X_{t+1+s}} \cdot d^{1-X_{t+1+s}} + \mu_{t+s+1} \left(\alpha(1-\tau_{t+s}) \cdot \hat{k}_{t+s+1}^{\alpha-1} + 1 - \delta \right) \right] = 0$$

The derivative with respect to μ_{t+s} gives the resource constraint (6.a). The equations above at s=0, are written by :

$$\mu_t = \hat{N}_t^{1-\sigma} \hat{c}_t^{1-\sigma}$$

$$\hat{c}_t^{-\sigma} = E_t \left[\beta (u^{X_{t+1}} d^{1-X_{t+1}})^{-\sigma} (1+n)^{-\sigma} \hat{c}_{t+1}^{-\sigma} \left\{ \alpha(1-\tau_{t+1}) \cdot \hat{k}_{t+1}^{\alpha-1} + 1 - \delta \right\} \right] \qquad (6.c)$$

Expression (6.c) is Euler's equation for the households' problem. It expresses the way in which instantaneous utilities vary, if we transfer a unit of consumption from period t to period t+1, while keeping the overall level of utility constant. The left-hand side expresses the loss of utility at time t due to the deprivation of one unit of consumption good. The right-hand side expresses the increase in expected utility due to the consumption of additional good at date *t*+1. In Euler's equation (6.c), the expected utility gain will depend on the government's future budget choice. If the government chooses to increase public spending, the additional gain expected from future consumption will increase. If the government chooses to reduce public spending, the future utility gain will be reduced.

However, it is necessary to determine whether there is a long-run steady state for consumption, production and efficient capital. A steady state for $\hat{c}_t$, $\hat{k}_t$, and $\hat{y}_t$ assumes that these variables become constant: $\hat{c}_t = \hat{c}$, $\hat{k}_t = \hat{k}$ et $\hat{y}_t = \hat{y}$. Recall that the household income tax rate is:

$$\tau_t = \frac{u^{X_t} \cdot d^{1-X_t}}{1 + \gamma_t} \tau_{t-1}$$

By expanding this expression for τ_t (replacing $1 + \gamma_t$ by $\frac{Y_t}{Y_{t-1}}$ and dividing this last term by $\hat{N}_t$), we obtain:

$$\tau_t = \frac{1}{1 + n} \cdot \frac{\hat{y}_{t-1}}{\hat{y}_t} \cdot \tau_{t-1} \tag{7.a}$$

At the steady state of efficient production,

$$\tau_t = \frac{1}{1 + n} \cdot \tau_{t-1} \tag{7.b}$$

τ_t is a decreasing sequence that converges to 0, as soon as the population growth rate is strictly positive. We discuss the existence of the steady state along two main axes, depending on whether $n = 0$ or $n > 0$.

The first axis is where $n = 0$ (the population growth rate is zero), in this case, the tax rate is constant[17].

$$\tau_t = \tau_{t-1} = \tau$$

By using Euler's equation (6.c), we deduce the value of the efficient capital at the steady state:

$$\hat{k} = \left\{ \left(\frac{1}{\beta[Pu^{-\sigma} + (1 - P)d^{-\sigma}]} + \delta - 1 \right) \frac{1}{\alpha(1 - \tau)} \right\}^{\frac{1}{\alpha - 1}} \tag{8.a}$$

The steady state of efficient production is deduced immediately:

$$\hat{y} = \hat{k}^{\alpha} \tag{8.b}$$

By taking the expectation of the resource constraint (6.a) at the steady state, we deduce:

$\hat{c} = (1 - \tau) \cdot \hat{y} - [Pu + (1 - P)d - (1 - \delta)] \cdot \hat{k}.$ when:$P \cdot u + (1 - P) \cdot d = 1$, then,

$$\hat{c} = (1 - \tau) \cdot \hat{y} - \delta \cdot \hat{k} \tag{8.c}$$

The equalities (8.a), (8.b) and (8.c) reflect the steady state of the efficient variables. Let's remember that:

$$\hat{k} = \frac{K_t}{\hat{N}_t} \text{ and then }, K_t = \hat{k} \cdot A_t.N_t$$

$\hat{k}$ is defined in equation (8.a). $N_t = 1, \forall t \geq 0,$, beacuse $n = 0$. This, we have:

$$K_t = \hat{k} \cdot u^{\sum_{j=1}^t X_j} \cdot d^{t - \sum_{j=1}^t X_j} \cdot G_0$$

The level[18] of capital depends on the history of government budget decisions. The same is true for consumption, production and public expenditure:

$$C_t = \hat{c} \cdot u^{\sum_{j=1}^t X_j} \cdot d^{t - \sum_{j=1}^t X_j} \cdot G_0$$

$$Y_t = \hat{y} \cdot u^{\sum_{j=1}^t X_j} \cdot d^{t - \sum_{j=1}^t X_j} \cdot G_0$$

$$G_t = \tau Y_t$$

Thus, consumption, capital and production will be at their highest level if each year the government chooses to increase public expenditure ($X_t = 1, \forall t \geq 1$). If, on the other hand, the government chooses each year to reduce its public expenditure($X_t = 0, \forall t \geq 1$), then consumption, capital and production will always decrease and will tend towards 0, in the long run. The economy will be in difficulty and the populations in misery.

The second axis of discussion is that where, $n > 0$, in this case, the steady state of efficient production requires that the tax rate on households' income be decreasing, and converge towards 0 when the horizon t grows to infity($\tau_t = \frac{1}{1+n} \cdot \tau_{t-1}$). Note that, even if τ_t tends towards 0 when t tends to infinity,$G_t = \tau_t \cdot Y_t$ does not tend to 0. Indeed, when there is a steady state for efficient production, then:

$$Y_t = \hat{y} \cdot (1 + n)^t \cdot u^{\sum_{j=1}^t X_j} \cdot d^{t - \sum_{j=1}^t X_j} \cdot G_0$$

and so:

$$G_t = \tau_t \cdot Y_t = \frac{\tau_0}{(1 + n)^t} \cdot \hat{y} \cdot (1 + n)^t \cdot u^{\sum_{j=1}^t X_j} \cdot d^{t - \sum_{j=1}^t X_j} \cdot G_0$$

$$G_t = \tau_0 \cdot \hat{y} \cdot u^{\sum_{j=1}^t X_j} \cdot d^{t - \sum_{j=1}^t X_j} \cdot G_0$$

Thus, G_t does not converge towards 0, when τ_t tends towards 0. By writing the Euler equation when the rate of tax pressure is zero, we obtain in the steady state of the efficient capital:

$$\hat{k} = \left\{ \left(\frac{1}{\beta[Pu^{-\sigma} + (1 - P)d^{-\sigma}] \cdot (1 + n)^{-\sigma}} + \delta - 1 \right) \frac{1}{\alpha} \right\}^{\frac{1}{\alpha - 1}} \tag{9.a}$$

As before, we deduce:

$$\hat{y} = \hat{k}^\alpha \quad and \quad \hat{c} = (1 - \tau) \cdot \hat{y} - [Pu + (1 - P)d - (1 - \delta)] \cdot \hat{k}.$$

We can also write the new values of K_t, C_t, Y_t, and G_t, when n>0 and τ_t tends towards 0 :

$$K_t = \hat{k} \cdot (1+n)^t \cdot u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$$

$$C_t = \hat{c} \cdot (1+n)^t u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$$

$$Y_t = \hat{y} \cdot (1+n)^t \cdot u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$$

$$G_t = \tau_0 \cdot \hat{y} \cdot u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$$

When the growth rate of the population is strictly greater than 0, capital, consumption and production all grow at the same rate[19] γ_t. γ_t is such that

$$1 + \gamma_t = (1+n) \cdot u^{X_t} \cdot d^{1-X_t}.$$

The growth rate τ_t is not constant, it depends at each date on the choice of the government, to increase or reduce its public expenditure. The growth rate of the aggregate variables also increases with the population growth rate. On the other hand, per capita consumption, per capita capital and per capita production depend on the history of government choices.

$$\frac{K_t}{N_t} = \hat{k} \cdot u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$$

$$\frac{C_t}{N_t} = \hat{c} \cdot u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$$

$$\frac{Y_t}{N_t} = \hat{y} \cdot u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$$

Capital per capita, consumption per capita and production per capita grow at the same rate $\tilde{\gamma}_t$; this rate $\tilde{\gamma}_t$ is such that: $1 + \tilde{\gamma}_t = u^{X_t} \cdot d^{1-X_t}$. All of these per capita variables increase if and only if the government chooses to increase public spending. Per capita consumption, per capita capital and per capita production decrease year by year if the government chooses each year to reduce public expenditure. The government's decision on public spending is the only source of per capita consumption growth.

If we assume that the sequence $(X_t)_{t\geq 1}$, of government's decisions, is a sequence of independent random variables, then it is possible to compute the expectation of capital, consumption and the production. Indeed, under the assumption of independence, $\sum_{j=1}^{t} X_j$ follows a binomial distribution[20] with parameters (t, P).

$$E\left(u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j}\right) = \sum_{j=0}^{t} C_t^j P^j (1-P)^{t-j} u^j d^{t-j}$$

$$E\left(u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j}\right) = \sum_{j=0}^{t} C_t^j (Pu)^j [(1-P)d]^{t-j}$$

$$E\left(u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j}\right) = [Pu + (1-P)d]^t.$$

$Pu + (1-P)d = E(u^{X_t} \cdot d^{1-X_t}) = E\left(\frac{G_t}{G_{t-1}}\right)$ is the expected growth rate of public

expenditure. Knowing the value of $E\left(u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j}\right)$, we can write:

$$E\left(\frac{C_t}{N_t}\right) = \hat{c} \cdot G_0 \cdot [Pu + (1-P)d]^t$$

If $Pu + (1-P)d = 1$, (the expected growth rate of public expenditure is zero), then expected consumption per capita is constant. The expected consumption per capita is decreasing if $Pu + (1-P)\, d < 1$ (when the expected growth rate of public expenditure is negative). The expected per capita consumption increases over time if $Pu + (1-P)d > 1$ (that is, when the expected growth rate of public expenditure is positive). The conclusions on capital per capita and production per capita are identical to those for consumption per capita.

It would be difficult to consider the case where the population growth rate is negative, $n < 0$. Indeed, $n < 0$ implies that the rate of tax pressure τ_t is strictly increasing and grows to infinity, when the horizon t becomes large. This situation implies that there is no steady state even for efficient variables. We exclude the possibility that the rate of fiscal pressure becomes infinitely large.

Conclusion

This paper provides an explanation of the sources of growth (or decline) and technological progress in fragile States. The paper identifies the government's decision (decision not known in advance) in public spending as the source of technology shocks. The descriptive analysis shows that the industrialized countries of the OECD are those where the levels of public expenditure per capita and income per capita are high. On the other hand, the developing countries of Africa and Asia are generally those where public expenditure and GDP per capita are low. Technological progress

or backwardness appears to be the result of public spending decisions. The increase in public spending boosts the growth rate of private sector production and households' consumption.?

Notes

1. Department For International Development.

2. Indeed, the factors of production (labour and capital) alone are not enough to explain economic growth. The "Solow" residuals are sometimes used to explain the share of growth unexplained by labour and capital. The Solow residual is the portion of an economy's output growth that cannot be attributed to the accumulation of capital and labor. the Solow (Solow 1957) residual is often described as a measure of productivity growth due to technological innovation. The Solow residual is also referred to as total factor productivity. However, total factor productivity is often used as a proxy for technological progress and innovation. Technological shocks are not very precise, and cover all the elements that improve the productivity of production factors. Among these elements, one can mention: the increasing in scientific knowledge, the increasing in the qualification of the workforce, more efficient technologies, better organization of production, etc.

3. Our main working hypothesis is that public spending explains economic growth. Public expenditures are a source of technological progress. An increase in public spending boosts the productivity of production factors (capital and labor) and generates economic growth.

4. Formulations (1), (3.a) and (3.b), assume that the government collects taxes and uses them to purchase the private good Y. The government budget is balanced, government revenue T_t, are such $T_t = G_t$ at each date t. G_t is the amount of government expenditure, it is the amount of private goods Y_t that the government buys. The government buys the goods produced by the private sector (roads, highways, airport, public electrification, drinking water, education, etc.) and gives them for free to private companies and households.

5. This formulation is taken from Baro (1990).

6. Equation (2) is important for developing countries, where the population growth rate is positive. In the work of Félix Atchadé (2018), sub-Saharan Africa has the fastest population growth in the world (2.7% per year over the period 2010-2015).

7. In the Barro model, $G_t = \tau \cdot Y_t$, the share of public expenditures in production is constant.

8. The parameters P, d and u can be estimated for each economy. We can estimate P by $\hat{P} = \frac{1}{T-1}\sum_{t=1}^{T} I_{G_t > G_{t-1}}$. $\hat{P}$ is the proportion of years in which public expenditure has increased. T is the number of observation years. $I_{G_t > G_{t-1}}$ is a random variable which takes the value 1 if $G_t > G_{t-1}$, and 0 otherwise. $\hat{u} = \frac{\sum_{t=1}^{T} \frac{G_t}{G_{t-1}} \cdot I_{G_t > G_{t-1}}}{\sum_{t=1}^{T} I_{G_t > G_{t-1}}}$ and $\hat{d} = \frac{\sum_{t=1}^{T} \frac{G_t}{G_{t-1}} \cdot I_{G_t \leq G_{t-1}}}{\sum_{t=1}^{T} I_{G_t \leq G_{t-1}}}$. $\hat{u}$(resp. $\hat{d}$) is the average magnitude of increasing (resp. reductions) in public spending.

9. The expression that describes the variable tax rate is given in next pages, $\tau_t = \frac{u^{x_t} \cdot d^{1-x_t}}{1+\gamma_t} \tau_{t-1}$; where $\gamma_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ is the growth rate of the output.

10. Uncertainty is modeled by a probability space (Ω, F, μ), where : $F = \{F_0 \subseteq F_1 \subseteq \cdots \subseteq F_t \subseteq F_{t+1}, \dots\}$, is a filtration of σ-algebras, ordered by inclusion, μ is a probability on the measurable space (Ω, F) ; F_t is precisely the σ-algebra generated by X^t; where, $X^t = \{X_0, X_1, X_2, \dots, X_t\}$ is the history of the shocks (hazardous and discretionary choices of the government) until the date t.

11. In neoclassical growth models, Cobb-Douglas production functions generally take the form: $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$, où $A_t = \exp(z_t)$, with : $z_t = \rho z_{t-1} + (1-\rho)z_{ss} + \varepsilon_t$. In this formulation, z_{ss}, σ^2 et ρ are constants parameters, and $(\varepsilon_t)_{t=0,1,2,\dots}$ is a sequence of random variable with mean 0 and standard deviation σ^2. The usual models tell us nothing about the origin of the shocks ε_t which explain technological progress, A_t. The notion of technological shocks or residuals in neoclassical model is not very clear. The origin of ε_t is unknown.

12. The expression of $A_t = u^{\sum_{j=1}^{t} X_j} \cdot d^{t-\sum_{j=1}^{t} X_j} \cdot G_0$, is one of the main contributions of this paper. The history of public spending explains technological progress or backwardness.

13. The principal Component Analysis is done with the SPAD software, version 5.5.

14. The data used can be found on the website: http://data.worldbank.org/data-catalog/world-development-indicators.

15. Just note that: $G_t = \tau_t \cdot Y_t = (u^{X_t} \cdot d^{1-X_t}) \cdot (\tau_{t-1} \cdot Y_{t-1})$

16. Strictly speaking, we should write $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$, so that when $\sigma = 1, U(C) = \ln(C)$. However, we should note that the choices of households remain invariant if we add a constant to their utility function.

17. This model says nothing about the value of the optimal tax rate. But, in a similar model, Barro (1990), finds that the optimal tax pressure rate is $\tau^* = 1 - \alpha$, where α is the capital intensity. Here, the tax pressure rate simply balances the government budget.

18. When population growth is zero, the aggregated variables are identical to the per capita variables. The discussion on the variables per capital is made a little further down.

19. The expression of $\gamma_t = (1 + n) \cdot u^{X_t} \cdot d^{1-X_t}$, shows that the only variables that explain the growth of capital, consumption and production in the long run are government decisions on public expenditure and population growth.

20. Where, $C_t^j = \frac{t!}{j!(t-j)!}$, with $j! = j \times (j-1) \cdots \times 1$, and $0! = 1$.

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